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Neutrino Capture Cross Sections for ^{40}Ar and β-decay of ^{40}Ti

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ABSTRACT

Shell-model calculations of solar neutrino absorption cross sections for ^{40}Ar, the proposed component of the ICARUS detector, are presented. It is found that low-lying Gamow-Teller transitions lead to a significant enhancement of the absorption rate over that expected from the Fermi transition between the isobaric analog states, leading to an overall absorption rate of 6.7 SNU. We also note that the pertinent Gamow-Teller transitions in ^{40}Ar are experimentally accessible from the β-decay of the mirror nucleus ^{40}Ti. Predictions for the branching ratios to states in ^{40}Sc are presented, and the theoretical halflife of 53 ms is found to be in good agreement with the experimental value of 56^{+18}_{-12} ms.

Subject headings: Nuclear Structure – Solar Neutrino Spectroscopy.

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The basic feature of solar neutrino astronomy is to provide a tool that permits the direct examination of processes that occur in the interior of the Sun. Neutrinos interact only via the weak interaction, and have essentially an infinite mean-free path in a normal stellar medium. Therefore, neutrinos observed on Earth probe solar processes that are occurring in the present, as opposed to photons, which emerge after $\approx 10^4$ yr.

Considerable interest has been generated by four solar neutrino experiments [**?**, **?**, **?**, **?**] that yield results that are different from those expected from the combined predictions of the standard solar model and the standard electroweak theory with zero neutrino masses. In essence, there seems to be a significant suppression in the neutrino flux over that predicted from the combined standard models. Two primary questions then remain: (1) do these experiments require new physics beyond the standard model for electroweak interactions, or (2) is the standard solar model at fault? A recent study [**?**] of over 1000 precise solar models concludes that it is not possible to simultaneously describe all four experiments within the framework of standard solar models, and suggests that physics beyond the standard electroweak theory is required. However, before definitive conclusions on the presence of new physics can be reached, further experimental verification is warranted. Indeed, three of the four experiments were sensitive to different parts of the solar neutrino spectrum, and it remains to be decided if the fluxes from all neutrino sources are suppressed, or if some mechanism suppresses higher-energy neutrinos, such as those from ^{8}B, so that the Ga experiments (GALLEX [**?**] and SAGE [**?**]) may be interpreted as detecting the full flux from the pp-chain.

One proposal to examine higher energy solar neutrinos is the Imaging of Cosmic and Rare Underground Signals (ICARUS) [**?**, **?**] experiment in which liquid argon (primarily ^{40}Ar) is the detector medium. As applied to solar neutrinos, ICARUS expects to measure the flux of solar ^{8}B neutrinos by both elastic scattering and absorption. In addition, by measuring the ratio between absorption events and elastic scattering events, which can be induced by all neutrino types, it is possible to deduce the probability of oscillations of electron neutrinos into μ and τ neutrinos independently of solar models [**?**]. Clearly, accurate knowledge of the

neutrino absorption cross section is needed. In the original proposal [?, ?], the feasibility of ^{40}Ar as a detector medium was assessed by assuming that neutrino absorption is dominated by Fermi transitions to the isobaric analog state (IAS) in ^{40}K. With this assumption (and the imposition of a 5 MeV cutoff on the minimum energy of the emitted electron), the solar neutrino absorption cross section is $\Sigma_{\rm tot}\,|_{\rm IAS}= 3.8\times10^{-43}$ cm^2, corresponding to a capture rate on ^{40}Ar nuclei $\mathcal{R}(^{40}{\rm Ar})\,|_{\rm IAS}= 2.2$ SNU. From the compilation of nuclear levels [?], however, we note that there are at least six $J^\pi = 1^+$, $T = 1$ levels in ^{40}K with excitation energies lower than the isobaric analog state. Given the dependence in the neutrino absorption cross section on the square of the energy of the emitted electron, these low-lying levels may contribute significantly to the overall neutrino absorption cross section.

Another branch of nuclear physics of great interest is the study of exotic nuclei, in which radioactive beams are increasingly used to study the properties of nuclei along the proton and neutron drip lines. Of particular interest to the ICARUS experiment is the β-decay of ^{40}Ti, which is the mirror of ^{40}Ar. Since isospin is a nearly conserved quantity (to within a few percent), the Gamow-Teller matrix elements pertinent to ^{40}Ar neutrino absorption can be determined experimentally from the branching ratios of the β-decay of ^{40}Ti to levels in ^{40}Sc (the mirror of ^{40}K). In addition, because of the large Q-value ($Q_\beta = 11.737$ MeV), all the transitions of interest in ^{40}Ar can be studied directly. The main difficulties that need to be overcome in this experiment are the short lifetime, measured to be $\tau_{1/2} = 56^{+18}_{-12}$ ms [?], and the fact that the β-decay occurs to states that are proton unbound in ^{40}Sc. The current experimental situation is that branching ratios have only been measured in which the decay proceeds via proton emission to the ground state of ^{39}Ca. Already at this stage, however, it can be deduced that Gamow-Teller transitions play an important role as the expected lifetime for ^{40}Ti in the limit of a pure Fermi transition is 157 ms. In addition, $20 \pm 4\%$ of all decays are found experimentally [?] to proceeded via β-decay to the second $J^\pi = 1^+$ state in ^{40}Sc (at 2.7 MeV) followed by proton emission to the ground state of ^{39}Ca.

In this paper, we present the results of a shell-model calculation for the Gamow-Teller transitions between the $J^\pi = 0^+$, $T = 2$ ground state of ^{40}Ar(Ti) to $J^\pi = 1^+$, $T = 1$ states in

^{40}K(Sc). We find that these low-lying Gamow-Teller transitions significantly enhance the solar neutrino absorption cross section, increasing the cross section over that from the isobaric analog transition by nearly a factor of three, namely up to $\Sigma_{\rm tot} = 11.5 \times 10^{-43}$ cm^2 and correspondingly $\mathcal{R}(^{40}\mathrm{Ar}) = 6.67$ SNU. For the purpose of comparison with future experiments, the branching ratios for the β-decay of ^{40}Ti are also presented. We note that the deduced half-life of 53 ms is in very good agreement with the experimental value of 56^{+18}_{-12} ms.

Nuclei lying across major shells, such as ^{40}Ar, pose a serious challenge to the shell model approach as two major oscillator shells must be included in the calculation, e.g. the $0\mathrm{d}_{5/2}1\mathrm{s}_{1/2}0\mathrm{d}_{3/2}$ (sd) and the $0\mathrm{f}_{7/2}1\mathrm{p}_{3/2}1\mathrm{p}_{1/2}0\mathrm{f}_{5/2}$ (fp) shells. Perhaps the most significant problem in performing calculations within this model space is the large number of configuration accessible. Indeed, it is not possible to carry out an unrestricted calculation for anything but the lightest nucleus in this model space. For this reason, a truncation on the model space must be imposed. Towards this end, we impose an $n\hbar\omega$ truncation, in which $n\hbar\omega$ denotes the excitation of n particles outside of the lower oscillator shell (in this case the sd-shell). A severe limitation, even within this approach, is the so-called "$n\hbar\omega$ truncation catastrophe". Since the expansion of the shell-model wave functions in an $n\hbar\omega$ model space converges slowly and the dimensions increase rapidly with n, one is often forced to use an effective interaction developed for use in the $0\hbar\omega$ space, which accounts for the gross properties of the rest of the series in an approximate way. Because of the slow convergence in n, the $n\hbar\omega$ catastrophe occurs even for $n = 2$. In essence, for mixed $(0+2)\hbar\omega$ calculations, the very strong interaction between the low-lying $0\hbar\omega$ and $2\hbar\omega$ states with similar symmetries causes the $0\hbar\omega$ states to be pushed considerably lower in energy, leaving an unrealistic gap in energy. In addition, not only are the binding energies grossly in error, but also the mixing between the 0 and $2\hbar\omega$ states is incorrect because it is dependent on the perturbed energies. Naturally, if dimensional considerations are not a concern, then one could at least partially solve the problem of the $2\hbar\omega$ states with the inclusion of $4\hbar\omega$ states.

Because of the problems inherent in an $n\hbar\omega$ truncation, we follow the example of ref. [?] and diagonalize the $0\hbar\omega$ and $2\hbar\omega$ spaces separately. In this light, the low-lying positive-parity

$T = 1$ and $T = 2$ states in $A = 40$ are purely $2\hbar\omega$ states, while the ground state of ^{40}Ca is the only $0\hbar\omega$ state. In addition, since the $2\hbar\omega$ states are not constructed by the excitation of particles of the same type out of the sd-shell, there are no spurious center-of-mass states in the calculation. The Hamiltonian used here is that of ref. [?], which consists of the Wildenthal matrix elements for the sd-shell [?], McGory's (0f,1p) shell Hamiltonian for the fp-shell matrix elements [?], and a modification of the Millener-Kurath potential for the cross-shell interaction [?]. For 25 nuclei with $Z = 13 - 20$ this interaction reproduced the ground-state binding energies with an rms deviation of 305 keV. In this work, the wave functions were computed using the shell-model program OXBASH [?] on VAX-4000/60 computers.

After diagonalizing the resulting Hamiltonian, and obtaining the wave functions, we calculate the neutrino capture cross section for the reaction

$$\nu \ + \ ^{40}\mathrm{Ar} \ \longrightarrow \ ^{40}\mathrm{K} \ + \ e^- \ , \tag{1}$$

for all states in ^{40}K with excitation energy up to the particle-decay threshold at 7.58 MeV [?]. The cross section for absorbing a neutrino with energy E_ν from the ground state of ^{40}Ar to the i^{th} excited state in ^{40}K is given by

$$\sigma_i(E_\nu) \quad = \quad \frac{G_V^2}{\pi c^3 \hbar^4} \ \mid \mathcal{M}_{o\rightarrow i} \mid^2 \, E_e^i \ k_e^i \ F(Z, E_e^i). \tag{2}$$

Here $F(Z, E_e^i)$ is the Fermi function associated with Coulomb correction factor appropriate for the charge density of the daughter nucleus, G_V is the vector coupling constant for nuclear weak processes, while k_e^i and E_e^i are the electron momentum and energy, respectively. They are given by

$$\begin{aligned} E_e^i \quad &= \quad E_\nu \ - \ Q_i \ + \ m_e c^2 \\ (k_e^i)^2 \quad &= \quad (E_e^i)^2 - m_e^2 c^4, \end{aligned} \tag{3}$$

with the Q–value being determined by the difference in binding energy for the initial and final states,

$$Q_i \ = \ E_i \ - \ E_o \ + \ m_e c^2. \tag{4}$$

The square of the transition matrix element $\mid \mathcal{M}_{o\to i} \mid^2$ is written as

$$\mid \mathcal{M}_{o\to i} \mid^2 = \left[\mathrm{B(F)}_{o\to i} + \mathrm{B(GT)}_{o\to i} \right], \tag{5}$$

where, in the long–wavelength limit, the Fermi and Gamow–Teller reduced transition probabilities are given by

$$\begin{aligned} \mathrm{B(F)}_{o\to i} &= \frac{1}{2J_o+1} \, |\langle J_i \,||\, t_\pm \,||\, J_o \rangle|^2 \\ &= \left[T_o(T_i+1) - T_{z\,o} T_{z\,i} \right] \, (1-\delta_C)\delta_{o,i}, \end{aligned} \tag{6}$$

and

$$\mathrm{B(GT)}_{o\to i} = \frac{1}{2J_o+1} \left(\frac{g_A}{g_V} \right)^2 \, |\langle J_i \,||\, (\vec{\sigma}\, t_\pm)_{\mathrm{eff}} \,||\, J_o \rangle|^2 . \tag{7}$$

The quantity $g_A/g_V = 1.2606 \pm 0.0075$ [?] is the ratio of the axial and vector weak coupling constants and δ_C is the isospin-mixing correction to the Fermi matrix element, which is of the order 0.2-0.4% [?], and can be neglected. In keeping with the observation that experimental B(GT) values are quenched relative to theoretical estimates, we have multiplied the free-nucleon Gamow-Teller operators by 0.775 [?].

The total absorption cross section for solar neutrinos is obtained by folding the cross section defined in Eq.(2) with the normalized solar neutrino flux $\Phi(E_\nu)$, and summing contributions due to excited states, i.e.

$$\Sigma_{\mathrm{tot}} = \sum_i \Sigma_i = \sum_i \int_{Q_i+E_{\mathrm{cut}}}^{\infty} \Phi(E_\nu)\; \sigma_i(E_\nu)\; dE_\nu . \tag{8}$$

The quantity $E_{\mathrm{cut}} = 4.489$ MeV is the minimum electron kinetic energy observable in the ICARUS detector, while the function $\Phi(E_\nu)$ associated with the ^{8}B spectrum was taken from ref. [?]. Multiplying Σ_{tot} by the total integrated neutrino flux $\mathcal{F}(^8\mathrm{B}) = 5.8 \times 10^6$ cm^{-2} s^{-1}, one obtains the neutrino capture rate on ^{40}Ar

$$\mathcal{R}(^{40}\mathrm{Ar}) = \sum_i \mathcal{R}_i(^{40}\mathrm{Ar}) = \Sigma_{\mathrm{tot}}(^{40}\mathrm{Ar})\; \mathcal{F}(^8\mathrm{B}). \tag{9}$$

For the β-decay of ^{40}Ti

$$^{40}\mathrm{Ti} \longrightarrow {}^{40}\mathrm{Sc} + e^+ + \nu , \tag{10}$$

the partial halflife for the decay to the i^{th} state in ^{40}Sc is given by

$$t^i_{1/2} = \frac{K}{G_V^2 \mid \mathcal{M}_{o\to i} \mid^2 f_{o\to i}\,(1+\delta_R)}, \tag{11}$$

where $K = 2\pi^3(\ln 2)\hbar^7/(m_e^5 c^4)$, and we use the value $K/G_V^2 = 6170 \pm 4$ s [?]. For the statistical rate function $f_{o\to i}$, we use the formalism of ref. [?], which is expected to be accurate to within 0.5%, namely

$$f_{o\to i} = \int_1^{W_0} \mathrm{d}W\, pW(W_0 - W)^2 F_0(Z,W)L_0(Z,W)C(Z,W)R(W) \tag{12}$$

where p and W are the electron momentum and energy, respectively, in units of $m_e c^2$, W_0 being the endpoint, and $F_0(Z,W)$, $L_0(Z,W)$, $C(Z,W)$, and $R(W)$ are parameterized correction factors given in ref. [?]. The radiative correction δ_R is given by ref. [?]

$$\delta_R = \frac{\alpha}{2\pi}\frac{\int \mathrm{d}W\, pW(W_0-W)^2 g(W,W_0)}{\int \mathrm{d}W\, pW(W_0-W)^2}, \tag{13}$$

where $g(W,W_0)$ is given by Eq. (III-21) in ref. [?]. The total halflife is then given by the sum of decay rates, i.e.

$$\frac{1}{t_{1/2}} = \sum_i \frac{1}{t^i_{1/2}}, \tag{14}$$

while the branching ratio to the i^{th} state is given by

$$\mathrm{BR}_i = \frac{t_{1/2}}{t^i_{1/2}}. \tag{15}$$

Shown in Table 1 are the explicit values for transitions to each of the $J^\pi = 1^+$, $T = 1$ levels in ^{40}K (^{40}Sc for the β-decay of ^{40}Ti). For the excitation energies, theoretical values are tabulated, as well as the experimental values for the first seven $J^\pi = 1^+$ levels as determined from the ^{40}K spectrum. One sees that there is a one-to-one correspondence between the theoretical and experimental levels, with the theoretical levels having an excitation energy of approximately 0.5–1.0 MeV higher than experiment. For the purpose of computing the neutrino cross sections and the β-decay partial half-lives, the experimental energies were used whenever possible. Also presented in the Table are the theoretical branching ratios for the β-decay of ^{40}Ti and their experimental values as deduced in ref. [?]. Note that the

experimental values are lower limits as they involve the decay process in which the proton-unbound excited state of ^{40}Sc decays directly to the ground state of ^{39}Ca. For completeness, the Table also reports the parameters for the Fermi transition to the isobaric analog state.

The primary conclusion of the shell-model calculation is that there is significant low-lying Gamow-Teller strength that leads to an overall enhancement of the neutrino absorption cross section by about a factor three over that expected from the Fermi transition alone. This conclusion is also supported by existing experimental data, where we find good agreement between the theoretical halflife $t_{1/2} = 53$ ms and the experimental value $\tau_{1/2} = 56^{+18}_{-12}$ ms [?]. In addition, the limited branching-ratio data indicates that most of the Gamow-Teller strength is to the second $J^{\pi} = 1^{+}$ state ($\geq 20\%$) as is predicted by theory.

The main drawback of the theoretical calculation, however, is that it is difficult to predict the magnitude of the uncertainty in the results. This is evident in Table 1 where the shell-model underpredicts the branching ratio to the first $J^{\pi} = 1^{+}$ state. To be noted, however, that even in complete shell-model calculations (i.e. without truncation and with a more sophisticated treatment of the quenching factors) the strengths obtained for the weakest states can show large deviations from their experimental values, while the strengths of the strongest states are usually more reliable. Aside from these uncertainties and keeping with the fact that one is trying to obtain reliable numbers to calibrate a high-energy neutrino detector, the most preferable course would be to determine the B(GT) values experimentally. Generally speaking, this is not possible since the nucleus of interest is usually stable. Instead, B(GT) values must be extracted from (n,p) or (p,n) reaction studies, or from the β-decay of the mirror nucleus by exploiting the fact that isospin is a conserved quantity to within a few percent. In fact, in the absence of the Coulomb potential (as well as the smaller nuclear charge-dependent interaction) and second-class weak currents, B(GT) values for β^{-} and β^{+} decays of mirror nuclei in the same isospin multiplet are identical. For the most part, in comparing the ft values of mirror analogs, the largest correction that needs to be applied is the effect due to different binding energies in the two analogs (cf. ref. [?] page 980 and ref. [?] page 107), while the corrections due to second-class currents are probably quite

small. Altogether, the experimental results for even-A nuclei in the range $8 \leq A \leq 30$ give a correction which appears to decrease in magnitude with increasing A, being of the order of 2–4% for the heavier cases [?]. This is, however, a point that deserves further careful study since no definite limits have been placed on second-class currents so far and since the calculation of the effects due to binding is also an open problem.

Because of the large Q-value for the β^+-decay of ^{40}Ti, it is in principle possible to measure all the B(GT) values of interest for neutrino absorption on ^{40}Ar directly. The primary difficulty in making this measurement is that ^{40}Ti β-decays into levels of ^{40}Sc that are proton unbound. In ref. [?], where the actual goal of the experiment was to observe direct two-proton radioactivity, only transitions involving proton emission to the ground state of ^{39}Ca were analyzed. As a result, not enough information exists to deduce experimental B(GT) values. Therefore, we strongly suggest that the experiment be repeated with a new focus. Namely, the *full* detection of β^+-transitions in ^{40}Ti to ^{40}Sc and their corresponding proton decays to ^{39}Ca, so that the branching ratios relevant to the β^+-decay of ^{40}Ti may be obtained. This course of action would, in point of fact, be of crucial importance for the ICARUS experiment in the context of the study of higher energy solar neutrinos, allowing an accurate calibration of the associated detector.

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TABLE 1 – Results of the shell-model calculation for the neutrino capture on ^{40}Ar and for the β-decay of ^{40}Ti. The available experimental values for E_i and BR$_i$, and the parameters for the Fermi transition (IAS) are also reported. The different quantities are explained in the text.

i	E_i(th) (MeV)	E_i(ex) (MeV)	$\mid \mathcal{M}_{o\to i} \mid^2$	$f_{o\to i}$	$t^i_{1/2}$ (sec)	BR$_i$(th) (%)	BR$_i$(ex) (%)	Σ_i (10^{-46} cm^2)	$\mathcal{R}_i(^{40}\mathrm{Ar})$ (SNU)
1	2.684	2.290	0.006	35455	29.0	0.18	4	20.21	0.012
2	2.971	2.730	1.195	27972	0.185	28.72	20	3262.28	1.892
3	3.291	3.110	0.946	21821	0.299	17.7	3	2075.09	1.204
4	3.622	3.146	0.101	21337	2.86	1.85	–	218.54	0.127
5	4.308	3.739	0.034	14524	12.1	0.44	–	50.68	0.029
6	4.521	3.798	1.119	13955	0.395	13.42	–	1643.01	0.953
IAS	–	4.384	4	9819	0.157	33.74	16	3847.32	2.23
7	4.801	4.789	0.082	6761	11.1	0.48	–	57.34	0.033
8	5.282	–	0.239	4511	5.72	0.93	–	106.13	0.062
9	5.642	–	0.030	3282	62.7	0.08	–	9.12	0.005
10	5.823	–	0.000	2774	∞	0	–	0	0
11	5.922	–	0.383	2524	6.38	0.83	–	81.35	0.047
12	6.151	–	0.023	2014	133.2	0.04	–	3.76	0.002
13	6.428	–	0.698	1510	5.85	0.91	–	76.15	0.044
14	6.480	–	0.343	1428	12.6	0.42	–	36.75	0.021
15	6.683	–	0.052	1141	104.0	0.05	–	4.11	0.002
16	6.876	–	0.017	912	398.0	0.01	–	0.97	0.001
17	7.087	–	0.121	705	72.3	0.07	–	4.77	0.003
18	7.123	–	0.002	674	4577	0	–	0.07	0
19	7.368	–	0.233	489	54.8	0.10	–	5.02	0.003